# Logic Gates

| Truth – 1 |
|---|
| False – 0 |

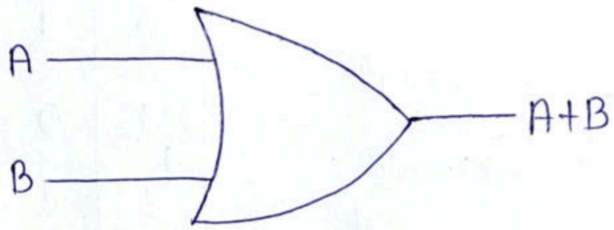
**Binary**

↳ 0, 1 digits

## OR Gate (+)

⇓

OR Gate gives you output as truth if one of the input is true

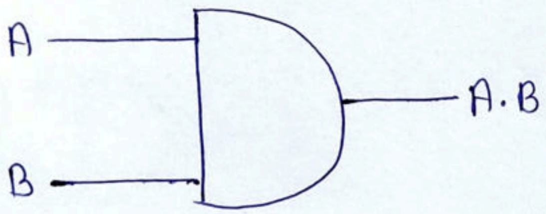



**Truth table**

| A | B | A+B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

## AND GATE

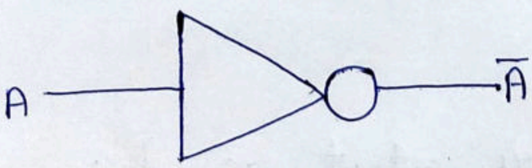
→ AND Gate give truth as output if both the inputs are trues.




| A | B | A.B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

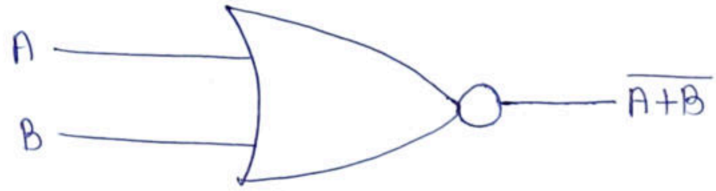
## NOT Gate ($^{-}$)

→ NOT Gate gives output to the opposite of input



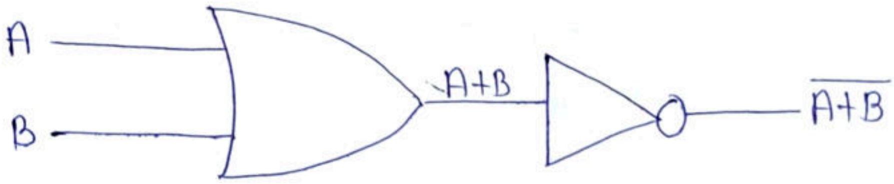

| A | $\bar{A}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

## NOR Gate → (NOT + OR) Gate

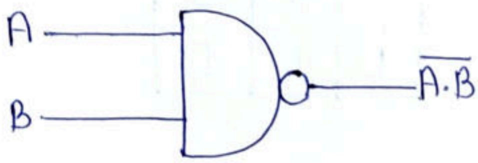



| A | B | A+B | $\overline{A+B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

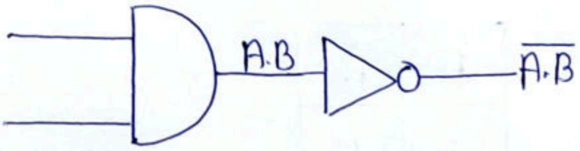
→ the NOR Gate is a digital logic gate that implements logical NOR – a true result (1) if both the inputs are false (0), otherwise output would be false.




## NAND Gate → (NOT + AND) Gate



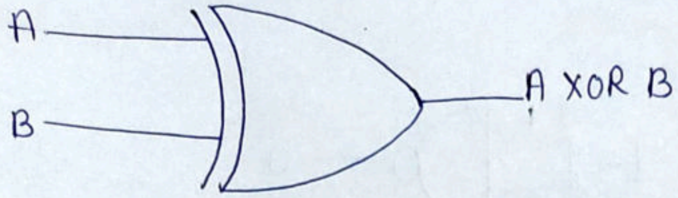

or




| A | B | A.B | $\overline{A \cdot B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

NAND Gate is one which gives you output as false only if both the inputs are true, otherwise output would be true

XOR Gate → (EXOR - Exclusive OR Gate)

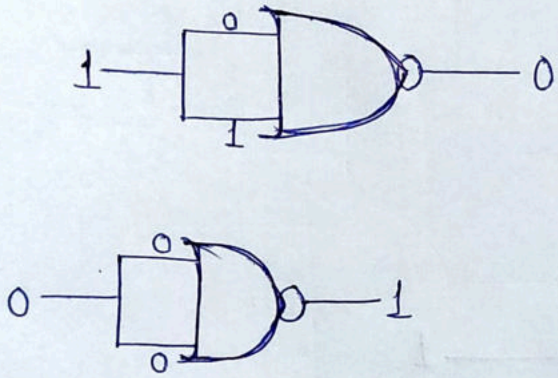



| A | B | A XOR B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

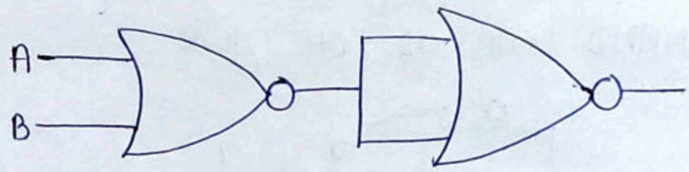
XOR Gate gives you output as false when both inputs are same either '0' or '1'. , otherwise Input would be true

☆ NOR Gate as universal gate

① NOR Gate as NOT

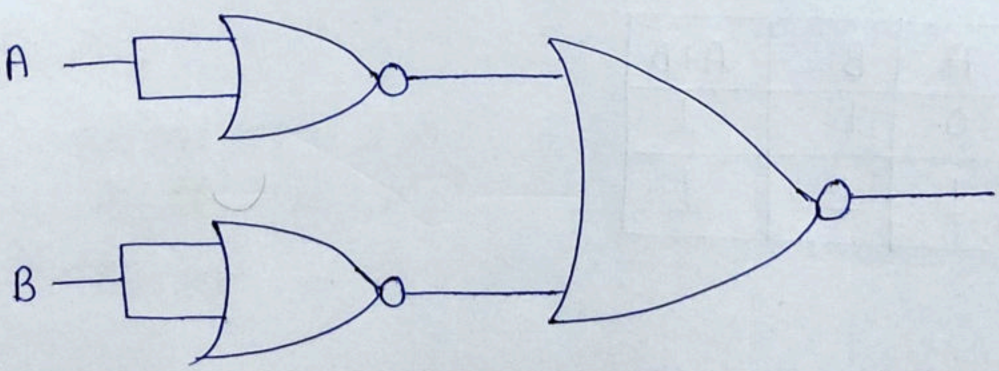



② NOR Gate as OR Gate




| A | B | A+B |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0 | 1 |

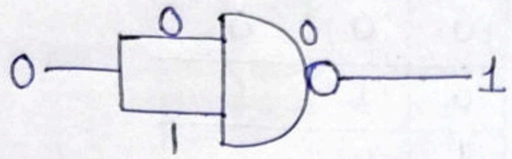
③ NOR Gate as AND Gate

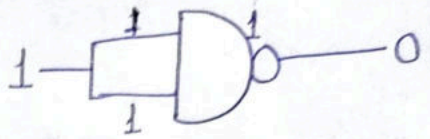



| A | B | A.B |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

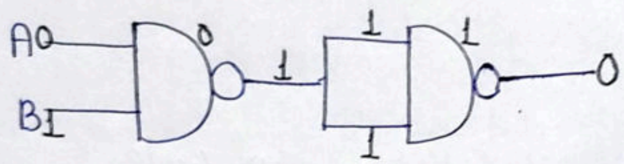
## ✯ NAND Gate as universal gate

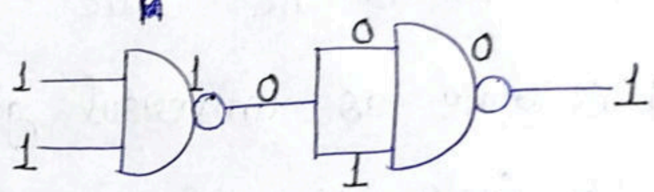
### ① NAND Gate as NOT Gate



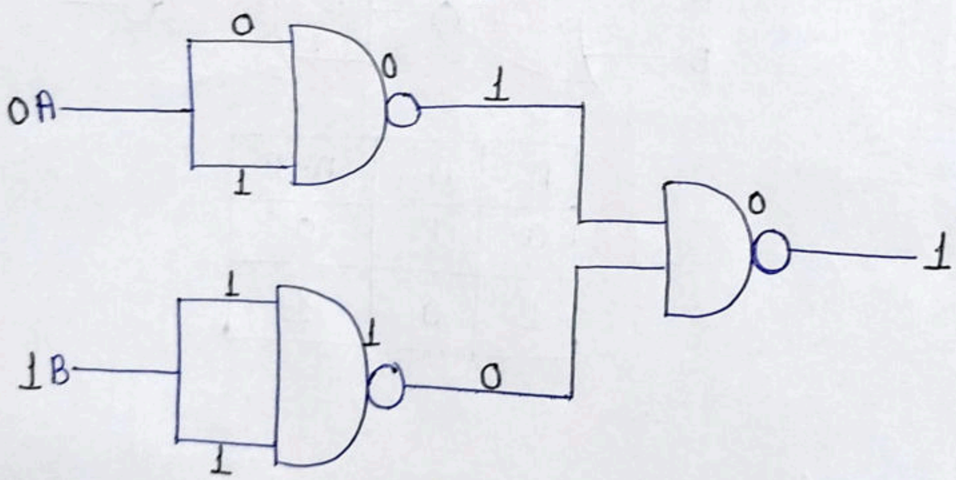


### ② NAND Gate as AND gate





### ③ NAND Gate as OR gate



| A | B | A+B |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 1 | 1 |

# Booleam Algebra

## Addition :

| Addition | Result | Carry |
|---|---|---|
| 0+0 | 0 | 0 |
| 0+1 | 1 | 0 |
| 1+0 | 1 | 0 |
| 1+1 | 0 | 1 |

**Ex:** 11+3 = 14

| | | |
|---|---|---|
| 2 | 11 | |
| 2 | 5 | 1 |
| 2 | 2 | 1 |
| | 1 | 0 |

| | | |
|---|---|---|
| 2 | 3 | |
| | 1 | 1 |

$$\begin{array}{r} \text{①}\ \ \text{①} \quad \\ 1\,0\,1\,1 \\ +\ \ 1\,1 \\ \hline 1\,1\,1\,0 \end{array}\ \text{Ans}$$

$\Rightarrow 1\ 1\ 1\ 0$
$\ \ \ 2^3\ 2^2\ 2^1\ 2^0$

$= 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 0$
$= 8 + 4 + 2$
$= 14$

**Ex:** 56 + 17 = ?

## Subtraction :

| subtraction | – | 0–0 | 0–1 | ① 1–0 | 1–1 |
|---|---|---|---|---|---|
| Result | – | 0 | 1 | 0 | 0 |
| Borrow | – | 0 | 1 | 0 | 0 |

**Ex:** 26 – 14 = ?

Soln:

| | | |
|---|---|---|
| 2 | 26 | |
| 2 | 13 | 0 |
| 2 | 6 | 1 |
| 2 | 3 | 0 |
| | 1 | 1 |

| | | |
|---|---|---|
| 2 | 14 | |
| 2 | 7 | 0 |
| 2 | 3 | 1 |
| | 1 | 1 |

$$\begin{array}{r} 1\,1\,0\,1\,0 \\ -\ \ 1\,1\,1\,0 \\ \hline 0\,1\,1\,0\,0 \end{array}\ \text{Ans}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 2^3 & 2^2 & 2^1 & 2^0 \\ & 8+4=12 & & \end{bmatrix}$$

## Multiplication :

| multiplication – | $0\times0$ | $0\times1$ | $1\times0$ | $1\times1$ |
|---|---|---|---|---|
| Result – | 0 | 0 | 0 | 1 |

**Ex:** $101 \times 111 = ?$

**Sol$^n$:**

```
      1 0 1
    x 1 1 1
    -------
    1 1 0 1
    1 0 1 x
 + 1 0 1 x x
 -----------
 1 0 0 0 1 1
```

Ans

**Ex:** $12 \times 7 = ?$

**Ex:** $14 \times 11 = ?$

## Binary Division :

**Ex:**

```
      011
 11 | 1011
     - 11
     ----
      1011
     - 11
     ----
      0101
      - 11
     ----
        10
```

**Ex:**

```
      011
 11 | 1001
     - 11
     ----
      1001
     - 11
     ----
       011
      - 11
     ----
        00
```

## Basic identities of Boolean Algebra :

1. $x + 0 = x$
2. $x + 1 = 1$
3. $x + x = x$
4. $x \cdot 1 = x$
5. $x \cdot 0 = 0$
6. $x \cdot x = x$
7. $x + \overline{x} = 1$
8. $x \cdot \overline{x} = 0$
9. $(\overline{\overline{x}}) = x$

## Basic laws :-

Commutativity - (10) $x + y = y + x$.

(11) $x \cdot y = y \cdot x$

Associativity - (12) $(x+y)+z = x+(y+z)$

(13) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$

Distributive - (14) $x \cdot (y+z) = x \cdot y + x \cdot z$

(15) $x + (y \cdot z) = (x+y) \cdot (x+z)$

Exo Simplify - $F(x,y,z) = \bar{x}yz + \bar{x}y\bar{z} + xz$

Solⁿ:

$$F(x,y,z) = \bar{x}yz + \bar{x}y\bar{z} + xz$$

$= \bar{x}(yz + y\bar{z}) + xz$ (by distributive law)

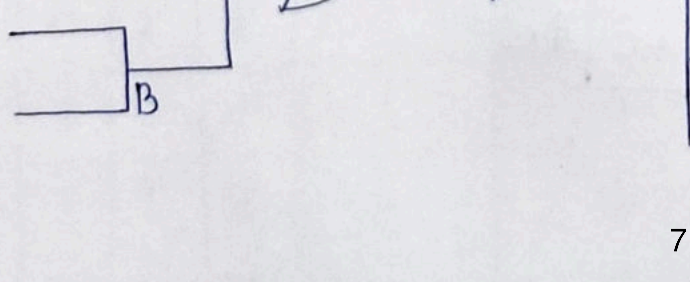
$= \bar{x}y(z+\bar{z}) + xz$ (by distributive law)

$= \bar{x}y \cdot 1 + xz$ (identity ; $x + \bar{x} = 1$)

$\boxed{F(x,y,z) = \bar{x}y + xz}$ Ans

## De-Morgan's Theorum -1

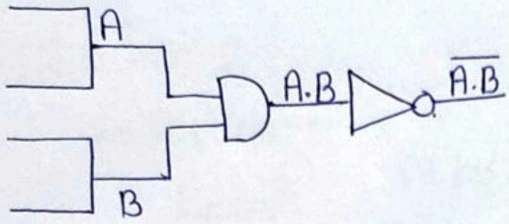
$$\overline{A+B} = \bar{A} \cdot \bar{B}$$




| A | B | A+B | $\overline{A+B}$ | $\bar{A}$ | $\bar{B}$ | $\bar{A}.\bar{B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

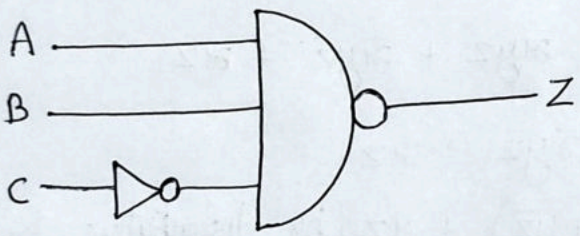
De-Morgan's Th. 2

$\overline{A \cdot B} = \overline{A} + \overline{B}$

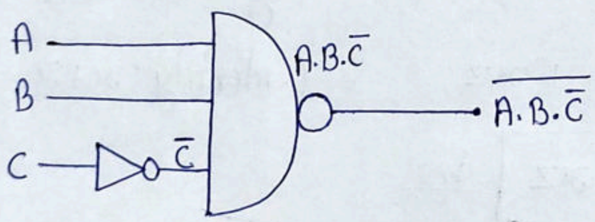



| A | B | AB | $\overline{A.B}$ | $\overline{A}$ | $\overline{B}$ | $\overline{A}+\overline{B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

**Exo** Determine the output expression for below circuit & simplify the output using De-Morgan's tho.




Solⁿ :




$F(A,B,C) = \overline{A.B.\overline{C}}$

Using de-morgan's tho. -

$F(A,B,C) = \overline{A.B.\overline{C}}$

$F(A,B,C) = \overline{A} + \overline{B} + \overline{\overline{C}}$

$\left[ \overline{A \cdot B} = \overline{A} + \overline{B} \right]$

$\boxed{F(A,B,C) = \overline{A} + \overline{B} + C}$ Ans

Exo Write the boolean ~~algebra~~ expression $z(y+z)(x+y+z)$ in the simplest form using boolean postolates. Verify your result by constructing truth table.

Soln,

$$F(x,y,z) = z(y+z)(x+y+z)$$

$$= (zy + z\cdot z)(x+y+z) \quad \text{(By distributive law)}$$

$$= (zy+z)(x+y+z) \quad (\text{identity} ; x\cdot x = x)$$

$$= (zy+z)\cdot x + (zy+z)\cdot y + (zy+z)\cdot z \quad \text{(distributive law)}$$

$$= zyx + zx + zy\cdot y + zy + zyz + zz$$

$$= xyz + xz + \underline{yz + yz} + yz + z \quad \left[\begin{array}{l}\text{commutative} \\ \text{idendity} ; x\cdot x = x\end{array}\right]$$

$$= xyz + xz + \underline{yz + yz} + z \quad (\text{identity} ; x+x = x)$$

$$\boxed{F(x,y,z) = xyz + xz + yz + z} \quad (\text{identity} ; x+x = x)$$

Truth table

| $x$ | $y$ | $z$ | $y+z$ | $x+y+z$ | $z(y+z)(x+y+z)$ | $xyz$ | $xz$ | $yz$ | $xyz+xz+yz+z$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |