TEST-2

1. a) i) Given : a = (135)(12) = (1235), b = (1579) to compute : $a^{-1}ba$, a^{-1} = (5321)
then $a^{-1}ba$ = (5321)(1579)(1235) = (1)(2)(3795)

ii) Klein y-group is

K_y = {l, I, j, k}

Whose multiplication table is

	l	i	j	k
l	l	i	j	k
i	i	l	k	j
j	j	k	l	i
k	k	J	i	i

We see that $i^2 = j^2 = k^2 = 1$

Since No element in the group has order equal to order of the group is 4

So, it is not cyclic.

(i) $x^n - 1 = 0$

$x^n = 1$, $x^n = (\cos 2\pi r + i\sin 2\pi r)$, r = 0, …., 1, n - 1

$x = (\cos 2\pi r + i\sin 2\pi r)^{1/4}$

$= \left(\cos\frac{2\pi r}{n} + i\sin\frac{2\pi r}{n}\right)$

$= \frac{e^{i2\pi r}}{n}$, r = 0, I, …. n - 1

Group $G = \left\{1, \frac{e^{i2\pi\times 1}}{n}, \frac{e^{i2\pi\times 2}}{n}, \ldots \frac{e^{i2\pi(n-1)}}{n}\right\}$

We see $G = \{\omega^{\circ}, \omega^1 \ldots \omega^{n-1}\}\left(\omega = e^{\frac{i2\pi}{n}}\right)$

We see that G is closed under multiplication and w not is the increase of wt and 1 is the identify and G is generated by ω

So G = <ω> and is cyclic

Since R is commutative using with unit element.

We have to prove that R is a division ring which proves R is a field.

Let O ≠ a Ɛ R be any non zero element

Let aR = {ar/reR}

Then we see that aR is a ideal.

By given condition aR = R or aR = {o}

But aR ≠ {O} as a ≠ O and a = a.i Ɛ aR.

Hence aR = R

Now [ƐR] 1 ƐaR ⇒ bƐR

S.t 1 = ab ⇒ b is right in relse of a

Thus R is a division Ring

Hence R is a field.

(i) $F(z) = 2^2$

Let $z = x + iy$

$F(z) = (x + iy)^2 = x^2 - y^2 + 2ixy$

Comparing it with u + iv

$u = x^2 - y^2$ and $v = 2xy$

$u_x = 2x, u_y = 2y, v_x = 2y, y_y = 2x$

$\boxed{u_x = v_y \text{ and } u_y = v_x}$

C-R equation is satisfied

∴ F(z) is analytic

(ii) $F(z) = 2xy + (x^2 - y^2)$

Hence $u = 2xy, v = x^2 - y^2$

$U_x = 2y, u_y = 2x, v_x = 2x, v_y = -2y$

$U_x \neq v_y$

So C – R equation is satisfied.

∴ **F(z) is not analytic.**

(iii) $\omega = \sin z$, let $z = x + iy$

$\omega = \sin(x + iy) = \sin x \cos iy + \cos x \sin iy$

$= \sin x \cosh hy + i\cos x \sinh y$

$u = \sin x \cosh y$ and $v = \cos x \sinh y$

$\frac{\int v}{\int x} = -\sin x \sinh y, \frac{\int v}{\int y} = \cos x \cosh y$

We see that $\frac{\int u}{\int x} = \frac{\int v}{\int y}$ *and* $\frac{\int v}{\int x} = \frac{\int u}{\int y}$

Hence u and v satisfy C-R equation

∴ ω is analytic.

(iv) F(z) = $\bar{z}$

Let z = x + iy, F(z) = x – iy

U = x and v = y

$u_x = 1, u_y = 0$ and $v_x = 0, v_y = 1$

$u_x = v_y$ and $u_y = -v_x$

∴ F(z) is analytic.

(d) Given $x^3 - ax + 1 = 0$

$F(x) = x^3 - 9x + 1 = 0$

Minimize Ƶ = $x_1 + x_2$

S.C $2x_1 + x_2 \geq 4$

$x_1 + x_2 \geq 0$

$x_1, ix_2 \geq 0$

If Max z = $-x_1 - x_2 + OS_1 + OS_2 - MA_1 - MA_2$

S.C $2x_1 + x_2 - s_1 + OS_2 + A_1 + OA_2 = 4$

$x_1 + 7x_2 + OS_1 - S_2 + OA_1 + A_2 = 7$

$x_1, x_2, s_1, z_2, A_1, A_2 \geq O$

Phase-I

	Cj	**0**	**0**	**0**	**0**	**-1**	**-1**		
CB	**Basic**	x_1	x_2	s_1	s_2	A_1	A_2	j	0
-1	A_1	2	1	-1	0	1	0	4	4/1
-1	A_2	1	(7)	0	-1	0	1	7	7/7=1
Zj = {CB CJ}		-3	-8	1	1	-1	-1		
Cj=Cj-Zj		+3	+8	-1	-1	0	0		

	Cj	**0**	**0**	**0**	**0**	**-1**	**-1**		
C_B	Basis	x_1	x_2	s_1	s_2	A_1	A_2	b	θ

-1	A_1	(13/7)	0	-1	1/7	1	-1/7	3	21/13
0	x_2	1/7	1	0	-1/2	0	1/7	1	
Zj		-13/17	0	1	-1/7	-1	1/7	-3	
Cj-Zj		13/17	0	-1	1/7	0	-1/7		

	Cj	**0**	**0**	**0**	**0**	**-1**	**-1**		
C_B	Basis	x_1	x_2	s_1	s_2	A_1	A_2	b	θ
0	x_1	1	0	-7/13	1/13	7/13	-1/13	21/13	
0	x_2	0	1	-1/3	-2/3	-1/13	-12/19	10/13	
Zj		0	0	0	0	0	0		
Cj-Zj		0	0	0	0	-1	-1		

All Cj S ≤ 0 and Max $z_1 = 0$

Phase II

	Cj	**-1**	**-1**	**0**	**0**	**b**
Basis	C_B	x_1	x_2	S_1	S_2	21/13
-1	x_1	1	0	-7/13	-1/13	21/13
-1	x_2	0	1	-1/13	2/23	
Zj		-1	-1	8/13	1/13	
Cj-Zj	0	0	-8/13	-13		

All Cj S ≤ 0

$\therefore (x_1, x_2) = \left(\frac{21}{13}, \frac{10}{13}\right)$ is

The optional solution

Max z = $\frac{-13}{13}$, ∴ Min z = $\frac{31}{13}$

2(a) Given R is a integral domain

Since Ch R = P, Px = 0 ∨ x ε R

Now $(a + b)^p = a^p - 1\ Pc_1\ a^{p-1}\ b \ldots PC_p\ b^p$ (as R is comutative)

We know that

P|PC$_r$, V$_r$, I ≤ r < p – 1

Thus each P$_{cr}$is a multiple of P.

Since a^{p-1}$_b$, a^{p-2} $_b^2$ … are all in p,

We find P$_c$, a^{p-1}b, Pc$_2$a^{p-2}b^2, are all zero.

Hence (a + b)^p = a^p + b^p.

(b) $\int_0^{2\pi} \frac{\sin^2\theta}{a+bcos\theta}$ do, a > b > 0.

We have $I = \int_0^{2\pi} \frac{\sin^2\theta}{a+bcos\theta}$ do

$$= \int_C \frac{\left\{\frac{1}{2}\left(2-\frac{1}{2}\right)^2\right\}}{\left\{a+\frac{1}{2}^2\left(2+\frac{1}{2}\right)\right\}} \frac{d_2}{i2}$$

(z = cosθ + isinθ)

$$\text{Sin}\theta = \frac{z+\frac{1}{2}}{2i}$$

$$\text{Cos}\theta = \frac{2+\frac{1}{2}}{z}$$

$$= \frac{l}{2i}\int_c \frac{(z^2-1)^2}{z^2(bz^2+2ab+b)} d_2$$

$$= \frac{-1}{2ib}\int_{cz^2} \frac{(2^2-1)^2}{\left[z^2+\frac{2a}{b}2+1\right]} d_2$$

$$= -\frac{1}{2i}\int_C F(z)d_2,\ say$$

Here C is a unit circle

F(z) has a pole of order two at z = 0 and poles at α, β where α, β are the roots of the $z^2 + \left(\frac{a}{b}\right)2 + 1 = 0$

Or $z = \frac{-a+\sqrt{a^2-b^2}}{b} = \alpha, z = \frac{-a-\sqrt{a^2-b^2}}{b} = \beta$

Since a, b > 0, |β| > 1, Also |αβ| = 1 there for |α|<|

Thus z = α is a simple Pole

And z = 0 are double pole inside c

Residence az = α.is $\underset{z\to\alpha}{lin}(z-\alpha)\left[-\frac{1}{2ib}\frac{(2^2-1)^2}{2^2(2-\alpha)(z-\beta)}\right]$

$$= \frac{-(\alpha-\beta)^2}{2ib(\alpha-\beta)}$$

$$= \frac{-1}{2ib}(\alpha - \beta) = \frac{-1}{2ib}.\frac{2}{b}\sqrt{a^2 - b^2}$$

$$= \frac{i}{b}\sqrt{a^2 - b^2}$$

Residue at z = 0 is

Coefficient of $\frac{1}{2}$ in $\frac{(z^2-1)^2}{2ibz^2\left(z^2+2\frac{a}{b}\right)^2+1}$

= coefficient of $\frac{1}{z}$ in $\frac{1}{2ibz^2}(1 - 2z + 24)\left(1 + \frac{2a}{b} + 2 + z^2\right)^{-1}$

$$= \frac{a}{ib^2} = \frac{-a^1}{v^2}$$

∴ $$\int_0^{2\pi} \frac{\sin^2\theta}{a+bcos\theta}\, do = 2\pi i \times \Sigma RT$$

$$= 2\pi i\left[\frac{i}{b}\sqrt{a^2 - b^2} - \frac{ai}{b^2}\right]$$

$$= \frac{2\pi i}{b^2}\left[a - \sqrt{a^2 - b^2} - \frac{ai}{b^2}\right]$$

(a) ox + 2y + z = 9

2x + 20y – 2z = –44

–2x + 3y + 10z = 22

$$x = \frac{1}{10}(9 - 2y - z)$$

$$y = \frac{1}{20}[-44 - 2x + 2z]$$

$$z = \frac{1}{10}[22 + 2x - 3y]$$

$$x^{k+1} + \frac{1}{10}[9 - 2y^k - z^k]$$

$$y^{k+1} = \frac{1}{20}[2z + 2x^{k+1} - 3y^{k+1}]$$

$x^\circ = 0$, $y^\circ = 0$, $z^\circ = 0$

$x_1 = 9$, $y_1 = -2.29$, $z_1 = 3.07$

$x_2 = 1.05$, $y_2 = -2$, $z_2 = 3.01$

$x_3 = 1$, $y_3 = -2$, $z_3 = 3$

(a) Let A = (q_0) be a maximal ideal $q_0 \neq 0$

Suppose $q_0 = 0$, then since R is not a filed, at least one $O \neq b \epsilon R$

$S + b^{-1}$ does not exist

Let B (b) and as $q_0 = 0$, A = (0)

And $O \leq B \leq R \geq A \leq B \leq R$

Now $B \neq A$ as $b \mathscr{E} B$, $b \neq 0$ and A = (0)

$B \neq R$ as $I \mathscr{E} R$ but $I \mathscr{E} B$

Note if $l \mathscr{E} B = (b)$ then ∃ some x $S + l = bx$ showing that b is invesible which is not be

Hence $a_0 \neq O$

(ii) q_0 is not a unit

Suppose q_0 is a unit then $q_0 q_0^{-1} = 1$

$q_0 \mathscr{E} A$, $q_0^{-1} \mathscr{E} R$ $|q_0 q_0^{-1} \mathscr{E} A$

$l \mathscr{E} R$

A = R

Which is not possible as A is maximal thus q_0 is not a unit

(iii) Let now $q_0 = bc$ for some b, $c \mathscr{E} R$, We show either b or c is a unit

Let B = (b)

Since $q_0 = bc$, $q_0 \mathscr{E} B$

⇒ all multiples of q_0 are in B

$A \leq B$

But A is maximal thus either B = R or B = A

If B = R, then $l \mathscr{E} B = (b)$ as $l \mathscr{E} R$

l = ab for some x

b is a unit

If B = a, then $b \mathscr{E} A = (q_0)$

$B = ya_0$ for some y

$q_0 = bc = yq_0C$

$q_0 - yq_0C = 0$

$q_0(1 - yc) = 0$

$1 - yc = 0$ as $(q_0 \neq 0)$

C is unit

Hence the result is proved.

Concisely, let q_0 be irreducible element

Let I be any ideal S.t $A < I \leq R$

Since R is a PID, J is generated by some

Element say x

Now $x \notin A$

Again, $A = (q_0) \leq I$

$Q_0 = xy$ for some y

Q_0 is ireducible of x or y is a unit

If y is a unit, then $yy^{-1} = 1$

and $q_0 = xy$ if $q_0 y^{-1} = x$

But $q_0 \in A$, $y^{-1} \in R$ $q_0 y^{-1} \in A$

$x \in A$, which is not true

Thus y is not a unit

So x is a unit and $xx^{-1} = 1$

Now $x \in I$, $x^{-1} \in R$, I is an ideal

$xx^{-1} \in I$ $\qquad$ $|\in I \mid I = R$

A is maximal ideal of R.

3(b) $\int_C \frac{z+y}{z^2+2z+5} d_z$, C is $|Z + 1 - i| = 2$

$F(z) = \frac{z+4}{z^2+2z+s}$

Poles of the equation F(z) is given by $z^2 + 2z + s = 0$

$$z = \frac{-2 \pm \sqrt{-lb}}{2} \Rightarrow z = \frac{-2 \pm 4i}{2}$$

$Z = -1 + 2i, -1 - 2i$

Given curve is a circle with center at (–1, 1) and radius 2

So only $z = -1 + 2i$ is inside the continuous.

Residue at $(Z = -1 + 2i) = \underset{z \to (-1+2i)}{lt} \frac{(2+1-2i)(Z+4)}{(Z+1+2i)(Z+1-2i)} = \frac{3+2i}{4i}$

According to Cauchy integral formula.

$\int_C \ F(z)d_2 = 2\pi i \sum R^+$

$\int_C \ \frac{z+y}{z^2+2z+3} = \frac{\pi}{2}(3+2i)$

$\int_0^1 \frac{x}{1+x} d_x$

Here n = 6, h = $\frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

	F(x)
$z_0 = 0$	0
$z_1 = \frac{1}{6}$	$\frac{2}{3}$
$z_2 = \frac{1}{3}$	$\frac{1}{2}$
$z_3 = \frac{1}{2}$	$\frac{2}{3}$
$z_4 = \frac{2}{3}$	$\frac{4}{5}$
$z_5 = \frac{5}{6}$	$\frac{10}{11}$
$z_6 = 1$	$\frac{1}{2}$

We know that $\int_0^1 \frac{x}{1+x} d_x = \frac{h}{2}[F(x_0) + 2\{F(x_1) + F(x_2) + F(x_3) + F(x_4) + F(x_5) + F(x_6)\}]$

$= \frac{1}{12}\left[0 + 2\left\{\frac{2}{7} + \frac{1}{2} + \frac{2}{3} + \frac{4}{5} + \frac{10}{11} + \frac{1}{2}\right\}\right]$

= .305226

$\simeq .305$

4(a) Let M be a maximal ideal of R.

Since R is Commutative ring with unity

$\frac{R}{M}$ is also a commutative ring with unity

Let $x + M \in \frac{R}{M}$ be anynon zero element

Then x + M ≠ M = ||x∉M

Let xR = $\{xr/r \in R\}$

It is easy to verify that xR is an ideal of R

So M + xR will be ideal of R.

x = 0 + x.l ∈ M + xR

M < M + xR ≤ R

M is maximal = M + xR = R

Thus |∈R =| | ∈ M + xR

l = m + xr for some m∈N, r∈R

l + M = (m + xr) + M

= (m + M) + (xr + M)

= (x + M)(r + M)

r + M is multiplication inverse of x + M

Hence $\frac{R}{M}$ is a field

Since $\frac{R}{M}$ is a field then $\frac{R}{M}$ is a integral domain.

Thus M is a prime ideal

(b) $$F(z) = \frac{z^2-1}{(Z+2)(Z+3)} = 1 - \frac{5Z+7}{(z+2)(z+3)}$$

Resolving into partial fractions, we get

$$F(z) = \frac{1+3}{z+2}\frac{-8}{z+3}$$

(i) 2 < |Z| < 3. Then $\frac{2}{|Z|} < 1 \; and \frac{|Z|}{3} < 1$

∴ $$\text{F(z)} = \frac{1+3}{z}\left(1+\frac{2}{z}\right)^{-1} - \frac{8}{3}\left(1+\frac{2}{3}\right)^{-1}$$

$$= 1 + \frac{3}{2}\left(1+\frac{2}{z}\right)^{-1} - \frac{8}{3}\left(1+\frac{2}{3}\right)^{-1}$$

$$= 1 + \frac{3}{2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n - \frac{8}{3}\sum_{n=0}^{\infty}(-1)\left(\frac{z}{3}\right)^n$$

$$= 1 + \frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n - \frac{8}{3}\sum_{n=0}^{\infty}(-1)\left(\frac{z}{3}\right)^n$$

(ii) $|z| > 3$ then $\frac{3}{|z|} < 1$

$$\therefore F(z) = 1 + \frac{3}{2}\left(1 + \frac{2}{z}\right)^{-1} - \frac{8}{z}\left(1 + \frac{3}{z}\right)^{-1}$$

$$= 1 + \frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n - \frac{8}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{3}{z}\right)^n$$

4(c) Total requirement = 340

Total availability = 340

∴ The given problem is balanced using vogel's Approximation method initial basic feasible solution is

4 (30)	1	2 (70)	6	9	100
6 (10)	4	3 (20)	5	7 (90)	120
5	2 (50)	6	4 (70)	8	120
40	50	70	90	90	

Now finding the value of 4i and y

As the maximum, no of basic cells must exist in the 2nd row

Putting $4_z = 0$, and finding the values of 4_i's and v_j's using.

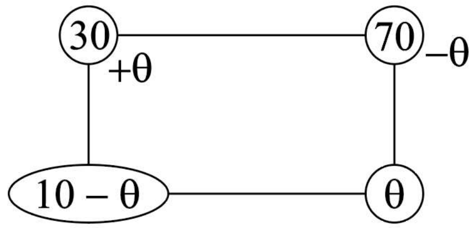
$\Delta_{ij} = 4_i + v_j - c_{ij}$

4 (30)	1 0	2 (70)	6 -ve	9 -ve	-2
6 (10)	4 –ve	3 (1)	5 (20)	7 (90)	0
5 (0)	2 (50)	6 -ve	4 (70)	8 -ve	-1
6	3	4	5	7	

Since the net evaluation in the cell (2, 3) is +ve therefore the current basic Feasible solution is not optional.

∴ the cell (2, 3) enters the basis.

We allocate the unknown quantity a loop involving basic cells around this entering cells.



Let $\theta = 10$, $x_{23} = 0$ (non-basic)

The cell (2, 3) l_0 areas the basis

The new basic feasible solutions is

4 (40)	1	2 (60)	6	9
6	4	3 (30)	5 (20)	-1 (90)
5	2 (50)	6	4 (70)	8

Again we calcuate u_i, v_j's

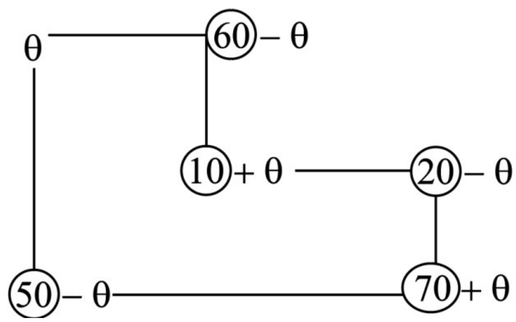
4 (40)	1 (1)	2 (60)	6 (-1)	9 (-1)	-1
6 (-1)	4 (-1)	3 (10)	5 (20)	7 (90)	0
5 -1	2 (50)	6 (-)	4 (70)	8 (-)	–1
5	3	3	5	7	

Since the net evaluation in the cell (1, 2) is +ve.

∴ the current basic feasible

Solution is not optional.

Making a closed 100P



Let θ = 20

The new basic feasible solutions is

4 (40)	1 (20)	2 (40)	6	9
6	4	3 (30)	5	7 (90)
5	2 (30)	6 0	4 (90)	8

Now again calculating u_i's and v_j's

4 (40)	1 (20)	2 (40)	6	9	0
6	4	3 (30)	5	7 (90)	1
5	2 (30)	6	4 (90)	8	1
4	1	2	3	6	

Since all $\Delta_{ij} \leq O$

∴ the current basic feasible solutions is optional.

The optional transportation cost = 1400.

5(a) Let h ∈ H, k ∈ K be any element

Then h ∈ H, k ∈ K ≤ 4, k ∈ K, is normal in 4.

Gives $(h^{-1})^{-1}$ k h^{-1} ∈ k l h^{-1} k h^{-1} ∈ k

K^{-1} h k h^{-1} ∈ k

K^{-1} h k h^{-1} ∈ H n k = (e)

hk = kh.

b(i) $\frac{Sinz-z}{z^3}$

we expand it

$$= \frac{\left(z-\frac{z^3}{3!}+\frac{z^5}{5!}\ldots\right)-z}{z^3}$$

$$= \left(\frac{1}{3!}-\frac{z^2}{5!}\ldots\right)$$

Since it doesnot have any principal term. So it has removalbe Singularity at z = 0.

$$\frac{cot\pi 2}{(z-a)^3} = \frac{cos\pi x}{\sin(\pi z)(z-a)^3}$$

Poles of F(z) use obtained by equaling to zero the denominator of F(x). Then we have

$(z-a)^3 \sin\pi z = 0$

∴ sinπz = 0 $0.(z-a)^3 = 0$

Now sinπz = 0 gives πz = nπ or z = n where n is any integal

And $(z-a)^3 = 0$ gives z = a

Hence z = a is a triple pole and

Z = 0, ±1, ±2 … are simple pole

Z = ∞ is a limit point of these simple pole therefore z = ∞ is non-isolated essential singulality.

(c) Here $u - v = \frac{cosx+sinx}{2cosx-e^4-e^{-4}}$

$$= \frac{1}{2}\left[\mathbf{1} + \frac{2cosa+2sinx-2e^{-y}}{2cosx-e^y-e^{-y}} - \mathbf{1}\right]$$

$$= \frac{1}{2}\left[\mathbf{1} + \frac{sinx+sinhy}{cosx-coshy}\right]$$

Now $\frac{\int u}{\int u} - \frac{\int v}{\int u} = \frac{1}{2}\left[cosu(cosx - coshy) + \frac{(sinx+sinhy)siny}{(cosx-coshy)^2}\right]$

$$= \frac{1}{2}\left[\mathbf{1} - \frac{cosxcoshy+sinxsinhy}{(cosx-coshy)^2}\right] \ldots(i)$$

Ans $\frac{\int u}{\int y} - \frac{\int v}{\int y} = \frac{1}{2}\left[\frac{coshycosx+sinhysinx-1}{(cosx-coshy)^2}\right]$

Or $\frac{\int y}{\int x} - \frac{\int u}{\int x} = \frac{1}{2}\left[\frac{coshycosx+sinhysinx-1}{(cosx-coshy)^2}\right]$ …(ii) (using C-R equation)

Solving (i) and (ii) we get

$$\frac{\int u}{\int x} = \frac{1}{2}\left[\frac{1-cosxcoshy}{(cosx-coshy)^2}\right] = \emptyset_1(x, y)$$

$$\text{ans } \frac{\int v}{\int x} = \frac{sinxsinhy}{2(cosx-coshy)^2}$$

∴ F(z) = $\frac{\int u}{\int x} + \frac{\int v}{\int x} = \emptyset_1(z, 0) + i\emptyset_2(z, 0)$

$= \frac{1}{2} \times \frac{1}{(1-cosz)}$

$= \frac{1}{4}(cosec^2)\frac{z}{2}$

∴ F(z) = $\frac{1}{4}\int cosec^2\frac{z}{2}d_2 + c$

$= -\frac{1}{2}\cot\frac{z}{2} + c$

At $z = \frac{\pi}{2}$, F(z) = 0

$\therefore C = F\left(\frac{\pi}{2}\right) + \frac{1}{4}\cot\frac{1}{4}$

$\therefore F(z) = \frac{1}{2}\left(1 - \cot\frac{z}{2}\right)$

(d) $x^3 - 6x + 4 = 0$

Let $F(x) = x^3 - 6x + 4$

$F(x) = 3x^2 - 6$

We know that

$$x_{n+1} = x_n - \frac{F(x_n)}{F(x_n)}$$

$x_0 = 0$

$x_1 = .667$

$x_2 = .7302$

$x_3 = .732$

$x_4 = .7321$

n	x_0	F(x_0)	F'(x_0)	x_1

1	0	4	-6	.667
2	.6667	.2963	-4.6667	.7302
3	.7302	.0083	-4.4006	.732
4	.732	0	-4.3923	.7321

5(e) According to the question.

Let x and y be number of quintal purchased by the company of A and B respectively then

Minimize Z = 200x + 400y

Subject to constrait

$$x + y \geq 200$$

$$.25x + .75y \geq 100$$

$$.1x + .2y \leq 35$$

$$x, y \geq 0$$

fig……………………

leauiced are a

End Points are (0, 175), (0, 133.33), (50, 150) and (100, 100)

Z(0, 175) = 70,000, Z(0,133.33) = 53,332

$\boldsymbol{Z_{(50,50)} = 70,000}$

$\boldsymbol{Z_{(100,100)} = 60000}$

So, minimum Z is 53,332

5(a)

Let < R, +, .> be the given using with unity then <R, +> is an additive abedian group. We denote it by R^+

Horn (R^+, R^+) is a ring with unity

Define a mapping F : R → Horn $g_r(x) = rx$, $x \in R^+$

Since g, $(x + y) = r(x + y) = rx + ry = g_r(x) + g_r(y)$

We find g_r is abonomorphism

Thus $g_r \in$ Horn (R^+, R^+)

Again $r_1 = r_2$

$r_1x = r_2x$ for all $x \in R^+$

$g_{r1}(x) = g_{r2}(x)$ for all x

$$F(r_1) = F(r_2)$$

Or that F is a well defined mapping

Again $F(r_1) = F(r_2)$

$$g_{r1} = g_{r2}$$

$g_{r1}(x) = g_{r2}(x)$ for all $x \in R^+$

$r_1x = r_2x$ for $x \in \&^+$

In Particular $r_1 - 1 = r_2.1$ as $1 \in \&^+$

$r_1 = r_2$

Or that F is one one

Again $F(r_1 + r_2) = g_{r1 + r2}$

$F(r_1) + F(r_2) = g_{r1} + g_{r2}$

Where $g_{r1 + r2}(x) = (r_1 + r_2)x = r_1x + r_2x = g_{r1}(x) + g_{r2}(x)$

$$= g_{r1} + g_{r2}$$

Or that $F(r_1 + r_2) = F(r_1) + F(r_2)$

Now $F(r_1r_2) = gr_1r_2$ and $F(r_1)\,F(r_2) = g_{r1}g_{r2}$

Where $g_{r1r2}(x) = (r_1r_2)x = r_1(r_2x)$

$$= g_{r1}\,(r_2d)$$

$$= g_{r1}(g_{r2}(x))$$

$$= (g_{r1}g_{r2})x \text{ For all x.}$$

$$g_{r1r2} = g_{r1}g_{r2}$$

Or that $F(r_1r_2) = F(r_1)F(r_2)$

F is a homomorphism

Hence an imbeeding maping

6(b) Minimize $z = x_1 - 3x_2 - 2x_3$

Subject to

$$3x_1 - x_2 + 2x_3 \le 7$$

$2x - 4x_2 \geq 12$

$-4x_1 + 3x_2 + 8x_3 = 10$

$x_1, x_2 \geq 0$ and x_3 is unrestricted

Let $x_3 = \boldsymbol{x_3^1 - x_3^{11}}$, $\boldsymbol{x_3^1}$ and $\boldsymbol{x_3^{11}} > 0$

Minimize $z = x_1 - 3x_2 - 2(\boldsymbol{x_3^1 - x_3^{11}})$

Subject to $3x_1 - x_2 + 2(\boldsymbol{x_3^1 - x_3^{11}}) \leq 7$

$2x_1 - 4x_2 \geq 12$

$-4x_1 + 3x_2 + 8(\boldsymbol{x_3^1 - x_3^{11}}) < 10$

$-4x_1 + 3x_2 + 8(\boldsymbol{x_3^1 - x_3^{11}}) \geq 10$

$x_1, x_2, \boldsymbol{x_3^1}, \boldsymbol{x_3^{11}} \geq 0$

Minimize $z = x_1 - 3x_2 - 2(\boldsymbol{x_3^1 - x_3^{11}})$

Subject to $-3x_1 + x_2 - 2(\boldsymbol{x_3^1 - x_3^{11}}) > -7$

$2x_1 - 4x_2 \geq 12$

$4x_1 - 3x_2 - 8(x_3^1 - x_3^{11}) > -10$

$-4x_1 + 3x_2 + 8(x_3^1 - x_3^{11}) \geq 10$

Its dual is

Maximize $Z = -7y_1 + 12y_2 - 10y_3^1 + 10y_3^{11}$

Subject to

$-3y_1 + 2y_2 + 4y_3^1 - 4y_3^{11} \leq 1$

$+y_1 + y_2 - 3y_3^1 + 3y_3^{11} \leq -3$

$-2y_1 - 8y_3^1 + 8y_3^{11} \leq -2$

$2y_1 + 8y_3^1 - 8y_3^{11} \leq 2$

$y_1, y_2, y_3^1, y_3^{11} \geq 0$

Let $y_3 = y_3^1 - y_3^{11}$

Then Maximize $Z = -7y_1 + 12y_2 - 10(y_3)$

Subject to $-3y_1 + 2y_2 + 4y_3 \leq 1$

$y_1 - 4y_2 - 3y_3 \leq -3$

$-2y_1 - 8y_2 = -2$

$y_1, y_2 \geq 0$ and y_3 is unrestricted

(7)

x	30°	35°	40°	45°	50°
F(x)=Sinx	.5000	.5736	.6428	.7071	.7660

x	F(x)	ΔF(x)	Δ^2F(x)	Δ^3F(x)	Δ^4F(x)
30°	.5000	.0736			
35°	.5736		-.0044		
40°	.6428		-.0049	-.0005	0
45°	.7071	.0589	-.0054	-.0005	
50°	.7660				

By newton forward interpolation formula

$$F(x) = F(x0) + P\Delta F(x0) + \frac{P(P-1)\Delta^2 F(x_0)}{2!} + \frac{P(P-1)(P-2)\Delta^3 F(x_0)}{3!}$$

$$= .5000 + .4 \times (.0736) + \frac{(.4)(.4-1)}{2} \times (.0044) + \frac{(.4)(.4-1)(.4-2)}{3!}(.0005)$$

$$= .5000 + .02944 + (.001056) \qquad \left[\text{here } P = \frac{32-30}{5} = .4\right]$$

= .5304768

7(a)

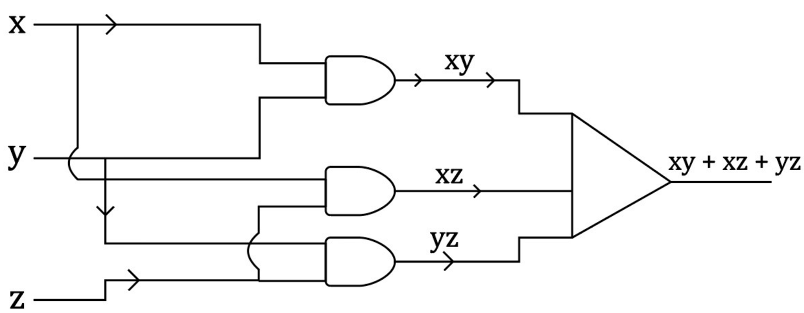
x	y	z	F(x, y, z)	
1	1	1	1 →	xyz
1	1	0	1 →	$xy\bar{z}$
1	0	1	1 →	$x\bar{y}z$
1	0	0	0 →	$\bar{x}yz$
0	1	1	1	
0	1	0	0	
0	0	1	0	
0	0	0	0	

Given $F(x, y, z) = xyz + xy\overline{z} + x\overline{y}z + \overline{x}yz$

$= xyz + xy\overline{z} + x\overline{y}z + xyz + \overline{x}yz + xyz$

$= xy(z + \overline{z}) + xz(\overline{y} + z) + yz(\overline{x} + x)$

$= xy + xz + yz$



$$\left|\frac{F(z)}{g(z)}\right| \leq \frac{64}{24+2+1}$$

$$|F(z)| > |g(z)|$$

$F(z)$ will have same root as

$$F(z) + g(z)$$

$F(z)$ has five roots inside $|z| = 2$.

So $F(z) + g(z)$ has five roots inside $|(z)| = 2$.

So in total there will be three roots inside the annulus $|\leq 12| < 2$.

(c)

	D_1	D_2	D_3	D_4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

First we convert it to minimization problem, we subtract each by largest value is 46 in this case.

(18) $\partial z^5 - 6z^2 + z + 1 = 0$

Let $F(2) = 2z^5$ and $g(z) = z - 6z^2 + 1$

Now on the circle $|z|=1$ we have

$$|F(z)|=|2z^5|=2$$

$$|g(z)|=|z-6z^2+1|\le|z|+|6z^2|+1\le 8$$

$$g(z)>F(z).$$

Thus $g(z)$ will have same root as

$$F(z)+g(z) \qquad \text{[Rouche theorem]}$$

$$g(z)=z-6z^2+1$$

or $\quad 6z^2-z-1=0 \qquad (3z+1)(2z+1)$

$$g(z)=z=\frac{1}{2},\frac{-1}{3}$$

So $F(2)+g(2)$ has two roots inside $\alpha z^5-6z^2+2=0$

For $|z|=2,$ Let $f(z)=\alpha z^5$ and $g(z)=-6z^2+2+1$

0	6	2	5
2	5	1	1
1	1	3	4
3	4	2	1

Subtracting minimum element from each row and column respectively we get

0	5	1	4
2	4	0	0
1	0	2	3
3	3	1	0

We need exactly four lines to cover all zero, hence optionality is reached.

$$A\to P_1, B\to D_3, C\to D_2, D\to D_y$$

Maximum profit = $16+15+15+15$

= 61

(a) $\quad 2x_1-x_2+3x_3+x_4=6$

$\quad 4x_1-2x_2-x_3+2x_4=10$

total number of solution is

$$4C_2 = 6.$$

$$Ax = b$$

$$A\begin{bmatrix} 2 & -1 & 3 & 1 \\ 4 & -2 & -1 & 2 \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, b = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

Since the rank of A is 2, the maximum number of linearly independent columns of A is 2.

Thus we consider any of the $\alpha \times 2$ sub-matrices as basic matrix B

$$\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}, \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

Let B = $\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}$

A basic solution to the system is obtained by taking $x_3 = x_4 = 0$ and solving the system.

$$\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

as $\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} = 0$, 1 B is not $Z - I$

If $B = \begin{bmatrix} -1 & 3 \\ -2 & -1 \end{bmatrix}$ then $x_1 = x_4 = 0$.

Solving $\begin{bmatrix} -1 & 3 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$

We get $x_3 = \frac{2}{7}$, $\quad x_2 = \frac{-36}{7}$

$\therefore \quad \left(0, \frac{-36}{7}, \frac{2}{7}, 0\right)^T$ is one of the basic solution.

If $B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $x_1 = x_2 = 0$

$$\therefore \quad \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

We get $x_3 = \frac{2}{7},\ x_4 = \frac{36}{7}$

$\therefore \quad \left(0,0,\frac{2}{7},\frac{36}{7}\right)^T$ is one of the basic solution.

Similarly $\left(\frac{18}{7},0,\frac{2}{7},0\right)^T$ is one of the basis solution and $\begin{bmatrix}-1 & 1\\ 2 & 2\end{bmatrix}, \begin{bmatrix}2 & 4\\ 4 & 2\end{bmatrix}$ are not L.I.

Thus $\left(0,0,\frac{2}{7},\frac{36}{7}\right)^T, \left(\frac{18}{7},0,\frac{2}{7},0\right)^T$ and $\left(0,\frac{-36}{7},\frac{2}{7},0\right)^T$ are the basic solution.

8(a) Let $y_1+2=x_1,\ y_2+1=x_2$ and $y_3+1=x_3$ then $y_3+1=x_3$ then

$$\text{Min z } = -6(y_1+2)-2(y_2+1)-5(y_3+1)$$

$$= -6y - 2y_1 - 5y_3 - 29 \qquad \text{------(1)}$$

S.C $\quad 2y_1 - 3y_2 + y_3 \le 10 \qquad$ _____(2)

$-4y_1 + y_2 + 10y_3 \le 20 \qquad$ _____(3)

$2y_1 + 2y_2 - 4y_3 \le 43 \qquad$ _____(4)

and $y_1, y_2, y_3 > 0$

max $z' = -\min z$

max $z' = 6y_1 + 2y_2 + 5y_3 + 29$

S.c $\quad 2y_1 - 3y_2 + y_3 + y_y + 0y_5 + 0y_6 = 10$

$-4y_1 + 4y_2 + 10y_3 + 0y_4 + y_5 + 0y_6 = 20$

$2y_1 + 2y_2 - 4y_3 + 0y_4 + +0y_5 + y_6 = 43$ d d

	C_j	6	2	5	0	0	0		
C_B	Basis	y_1	y_2	y_3	y_4	y_5	y_6	b	θ
0	y_4	2	–3	1	1	0	0	10	$\frac{10}{2}$
0	y_3	–4	4	10	0	1	0	20	---
6	66	2	2	–4	0	0	1	43	$\frac{43}{2}$
Z_j		0	0	0	0	0	0		

$C_j - z_j$	6	2	5	0	0	0

	C_j	6	2	5	0	0	0		
C_B	Basis	y_1	y_2	y_3	y_4	y_5	y_6	b	θ
6	y_1	1	$\frac{-3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	50	---
0	y_5	0	–2	12	2	1	0	40	---
0	y_6	0	5	–5	–1	0	1	33	$\frac{33}{5}$
Z_j		6	– 4	3	3	0	0		
$C_j - z_j$		0	11	2	–3	0	0		

	C_j	6	2	5	0	0	0		
C_B	Basis	y_1	y_2	y_3	y_4	y_5	y_6	b	θ
6	y_1	1	0	– 1	$\frac{1}{5}$	0	$\frac{3}{10}$	$\frac{149}{10}$	$\frac{149}{10}$
0	y_5	0	0	10	$\frac{12}{5}$	1	$\frac{2}{5}$	266	266
Z	y_2	0	1	–1	$\frac{-1}{5}$	0	$\frac{1}{5}$	$\frac{33}{5}$	$\frac{33}{5}$
Z_j		6	2	– 3	$\frac{4}{5}$	0	$\frac{22}{10}$		
$C_j - z_j$		0	0	13	$\frac{-4}{3}$	0	$\frac{-22}{10}$		

	C_j	6	2	5	0	0	0		
C_B	Basis	y_1	y_2	y_3	y_4	y_5	y_6	b	θ
6	y_1	1	0	0	$\frac{22}{50}$	$\frac{1}{10}$	$\frac{17}{50}$	$\frac{1952}{100}$	$\frac{149}{10}$
5	y_3	0	0	1	$\frac{12}{50}$	$\frac{1}{10}$	$\frac{2}{50}$	$\frac{33}{25}$	266
2	y_2	0	1	0	$\frac{2}{50}$	$\frac{1}{10}$	$\frac{12}{50}$	$\frac{33}{5}$	$\frac{298}{25}$
Z_j		6	2	5	$\frac{196}{50}$	$\frac{13}{10}$	$\frac{136}{50}$		

$C_j - z_j$ 0   0   0 $\frac{-98}{25}$ $\frac{-13}{10}$ $\frac{-68}{50}$

$\therefore \quad \Delta y \le 0$

$\therefore \quad y_1 = \frac{976}{50}, \quad y_2 = \frac{298}{25}, \quad y_3 = \frac{133}{25}$

$x_1 = \frac{1076}{50}, \quad x_2 = \frac{323}{25}, \quad x_3 = \frac{208}{25}$

$\therefore \quad \max z' = 6y_1 + 2y_2 + 5y_3 + 29$

$= 196.56$

(b) Define a map $\theta : \frac{G}{k} \to G^1$ S.t

$\theta(Ka) = F(a), a\varepsilon 4$

to show that θ is an isomorphism.

<u>θ is well defined</u>

$Ka = kb$

$\Rightarrow \quad ab^{-1}\varepsilon k = kei\widehat{T}$

$\Rightarrow \quad F(ab^{-1}) = e^1$

$\Rightarrow \quad F(a) = F(b)$

$\Rightarrow \quad \phi(ka) = \phi(kb)$

<u>θ is 1^{-1}</u>

$\theta(ka) = \theta(kb)$

$F(a) = F(b)$

$ab^{-1}\varepsilon K$

$Ka = Kb$

<u>Q is homomorphism</u>

Let $g\varepsilon G^1$ be any element. Since $F : G \to G^1$ is onto $\exists g \varepsilon G$ s.t

$F(g_1 = g')$

Now $\theta\left(kg1 = F(g) = g^1\right)$

Hence θ is an isomorphism.

$F(g) = g^1$

Now $\theta\left(kg^1 = F(g) = g^1\right)$

Hence θ is an isomorphism.

(c) $\frac{dy}{dx} = x + y^2$.

Given that $F(x, y) = x + y^2$

Here we take h = – 1 and carry out the calculations in two steps.

Step - 1 $x_0 = 0, y_0 = 1, \ h = -1$

$K_1 = hF(x_0, y_0) = -1$

$K_2 = hF\left(x_0 + \frac{n}{2}, y_0 + \frac{k_1}{2}\right) = .1152$

$K_3 = hF\left(x_0 + \frac{n}{2} + y_0 + \frac{k_2}{2}\right) = .1168$

$K_4 = hF(x_0 + h_1 y_0 + k_3) = .1347$

$K = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = .1165$

$\therefore \quad y(0-1) = y_0 + k = 1.1165$

$x_1 = x_0 + n = .1, \qquad y_1 = .1165, h = .1$

$k_1 = hF(x, y) = .1347$

$k_2 = hF\left(x_1 + \frac{h_1}{2} + y_1 + \frac{1}{2}k_1\right)$

$= .1531$

$k_3 = hF\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = .1516$

$k_4 = hF\left(x_1 + h_1, y_1 + k_3\right) = .1823$

$k = \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) = .1571$

Hence $y(.2) = y_1 + k =$ 1.2736