# Born's Probabilistic Interpretation:-

The solution of the Schrödinger's equation,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

is the wavefunction $\Psi(x,t)$.

Notice that there is an $i$ in the eqn, which means the wavefunction is complex!

Even for free particles, the wavefunction solution $e^{i(kx-\omega t)}$ is a complex solutn.

So, the ~~ques~~ question arises, what is the wavefunction? After all the wavefunction is the quantity that describes the matter waves associated with the motion of particles. So, what is waving?

When we talk about the waves on a string, we are talking of the displacement of the atoms in the string from their equilibrium.

When we talk about light as an electromagnetic wave, we are talking about the oscillations of electric & magnetic fields.

You see, the wavefunction being complex ~~has~~ is a good thing, since wavefunction $\Psi(x,t)$ itself has no direct physical significance.

So then what is the point of solving the SE, if its solution is not physical?

After all, in classical mechanics, when we solve the Newton's 2nd law, $\vec{F} = m\frac{d^2\vec{r}}{dt^2}$, we obtain the solution $\vec{r}(t)$ which gives us the position of the particle at any time $t$, & from which we can find out other quantities like $v$, $p$, KE, trajectory etc.

You see, $\Psi(x,t)$ itself is not a physical quantity, but it contains information about the particle's position, momtm, energy etc that we can find out by performing certain mathematical operations.

So, the wavefunction is a computational device, that has significance only in the context of Schrödinger's theory.

One of its significance comes from the -

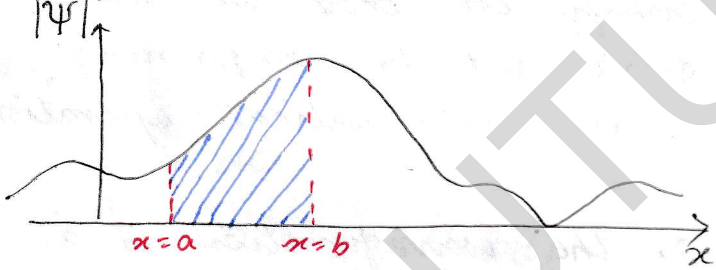
Born's Statistical Interpretation of the wave function, which says that $|\Psi(x,t)|^2 = \Psi^*\Psi$ gives the probability density of finding the particle

(where $\Psi^*$ is the complex conjugate of $\Psi$)

This was stated by Max Born in 1926. So the probability density $|\Psi(x,t)|^2 = \Psi^*\Psi$ gives us a likelihood of where the particle will be found, for a system.

→ If at any instant $t$, a measurement is made to locate the particle associated with the wavefn $\Psi(x,t)$, then the probability that the particle will be found b/w $x=a$ and $x=b$ is

$$P_{ab} = \int_{x=a}^{x=b} \Psi^*(x,t)\,\Psi(x,t)\,dx$$




This is in sharp contrast to classical mechanics where we can (once all forces and initial conditions are known) predict with certainty the future position of a particle.

Quantum mechanics only offers us – probability. So this statistical interpretation introduces an indeterminacy into QM.

Classical Mechanics ⟶ Deterministic

Quantum Mechanics ⟶ Indeterministic / Probabilistic

For, even if we know everything the QM theory has to tell about the particle, still we cannot predict with certainty the outcome of a simple experiment – to measure its position. All QM offers us is a statistical information about all possible ~~re~~ results.

This indeterminacy has been profoundly disturbing to physicists & philosophers alike, and it is natural to wonder whether the QM theory is incomplete, or nature actually behaves like this.

Keep in mind however, that there is a big difference b/w the probability of an event & the event itself. The wavefunction of a particle can be spread out in space, but it doesn't mean that the particle itself is spread out. (There's no such thing as 20% of the electron here, and 80% somewhere else)

→ The dividing line b/w the wave & the particle nature – is the act of measurement. In a way, everything in the future is a wave, everything in the past is a particle.

Now, you may ask, what inspired Born to give his statistical interpretation of the wavefunction.

According to Born, he was inspired by Einstein's idea of the photoelectric phenomenon — where the square of the optical wave amplitudes ~~probab~~ provide a probability density for the occurrence of photons

$|\varepsilon|^2 = I$ or $E = nh\nu$

wave amplitude — no. of photons

This concept can be carried forward to the psi-function $|\Psi(x,t)|^2$ to represent the probability density for electrons.

This also explains the interference pattern in the double slit experiment.


when $S_1$ is open $P_1 = |\Psi_1|^2$

when $S_2$ is open $P_2 = |\Psi_2|^2$

when $S_1$ & $S_2$ are open $P = |\Psi_1 + \Psi_2|^2$

① when $S_1$ is open

$$P_1 = \Psi_1^* \Psi_1 = |\Psi_1|^2$$

② when $S_2$ is open

$$P_2 = \Psi_2^* \Psi_2 = |\Psi_2|^2$$

③ when both $S_1$ and $S_2$ are open

$$P \neq |\Psi_1|^2 + |\Psi_2|^2$$

But

$$\begin{aligned} P &= |\Psi_1 + \Psi_2|^2 \\ &= (\Psi_1^* + \Psi_2^*)(\Psi_1 + \Psi_2) \\ &= \Psi_1^*\Psi_1 + \Psi_1^*\Psi_2 + \Psi_2^*\Psi_1 + \Psi_2^*\Psi_2 \\ &= |\Psi_1|^2 + |\Psi_2|^2 + \Psi_1^*\Psi_2 + \Psi_2^*\Psi_1 \\ &= P_1 + P_2 + \text{oscillating term} \end{aligned}$$

explains the interference pattern for electrons.

Thus the Born's statistical interpretation sets the stage for correctly interpretating the Schrödinger's equation.